HW 5

1.

Using the equivalence construction, I was able to convert the CFG from HW 4 question 1 to the PDA below.

Note: A language is context-free iff some pushdown automaton recognizes it  
  
 **Converted PDA**

Diagram

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2. To convert the PDA from HW 4 question 2 to a CFG we must follow the guidelines below:

Prerequisites for the PDA to CFG conversion P = (Q, Σ, Γ, δ, q0, {qaccept}):

1. Single accept state

2. Empties stack before accepting

3. Each transition either pushes one symbol to the stack, or pops one symbol off the stack, but not both or none.

My original PDA from HW 4 is shown below and does not satisfy these conditions to I had to tweak it to make it able to follow the rules to be able to produce a CFG from it. I had multiple final states, and my stack did not always empty when reaching the final state.  
  
**Original PDA**Diagram

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After making the changes I was able to formulate the new PDA below:

**PDA that satisfies requirements**

Table

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I also tested the new PDA to make sure it still accepted and rejected the same strings to verify the language is being followed. The test was a success, and I was able to now create a CFG from the PDA.

To show two rules for each type of the CFG that is equivalent to a PDA that satisfies the requirements for conversion I listed them below:

Type1 Rules: Aq0,q0 -> E , Aq1,q1 -> E

Type2 Rules: Aq0,q2 -> Aq0,1Aq1,q2 , Aq1,q7 -> Aq1,q3Aq3,q7

Type3 Rules: Aq0,q7 -> Aq1,q3 , Aq0,q7 -> Aq8,q2$

3.   
(a) reverse  
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(b) max(L) = {w ∈ L | wu ∈/ L for any u =/ ∈}

We let M = (Q, **Γ, q0, Z, F, δ) be a PDA which recognizes w where w is some context-free language.**

**We assume that the PDA for language max accepts that only the strings reaching the final state but not those strings that are added to reach a final state again.   
Therefore, the strings exactly ending in final states are accepted.**

**For a state q** ∈ **F, check whether there is a path from q** ∈ **Q to any state in F using DFS. Let F’ F be the set of all states from which there is no path. Now changing the set of final states F to F’ gives a PDA for the language.**

**We already know that** a language is context-free if and only if some pushdown automaton recognizes it.

**Therefore, we have proven that Max(L) is a context-free language since we have constructed a PDA for the language.**

Text, letter

Description automatically generated4. My example of a context free language whose comlement is not context free is the pair: L = {ww | w ∈ {a,b}\* and L1 = {a,b}\* \ L1  
My proof is below  
  
  
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\*\*Above when I mentioned L2 in the second image I was referring to L1 sorry for the confusion\*\*

5.

I will use the previous proof to prove that the given language is not context-free using pumping lemma, the work is shown below:

